Malla 1. Constant

20 November 2023

(It's on the course website calendar.)

- Today: Quiz 4 matrix calculations
- 11 December: Midterm exam 🎉 0
- January: Quiz 5 and 6
- February: Final exam *(and optional retake)*. 0





The $n \times n$ identity matrix, written $I_{n \times n}$ or I_n or just I, is a special matrix such that, if the products exist, • $I\vec{v} = \vec{v}$ for any vector \vec{v} ,

- IM = M for any matrix M,
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In a way, each matrix $I_{n \times n}$ acts like the number 1 because multiplying by it does not cause any change.

Formulas: $I_{2\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. $I_{3\times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. $I_{4\times 4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.





The inverse of M, written M^{-1} , is the matrix for which $MM^{-1} = M^{-1}M = I$. For a 2 × 2 matrix $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, we have $M^{-1} = \frac{1}{\det t}$

where

det(M)

is called the determinant of M.

A square matrix has an inverse if and only if its determinant is not zero.

For larger matrices the formulas are worse, but the boxed fact is still true.

$$\frac{1}{(M)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix},$$

$$ad - bc$$

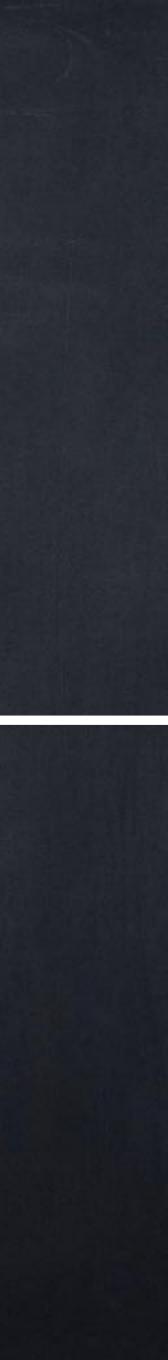


Arichmelic

Creometry

Algebra

Analysis



Arichmelic

Task: Divide 30 by 12.

Answer: $\frac{5}{2}$ or $2+\frac{1}{2}$ or 2.5or 2 remainder 6

You need to be comfortable with calculations before solving equations.

Algebra

Task: Solve 12x = 30.

Answer: $x = \frac{5}{2}$ Or...

Arichmelic

Task: Multiply $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\begin{bmatrix} 5 \\ 6 \end{bmatrix}$

matrix You need to be comfortable with calculations before solving equations.

Algebra

Task: Solve $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} X = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

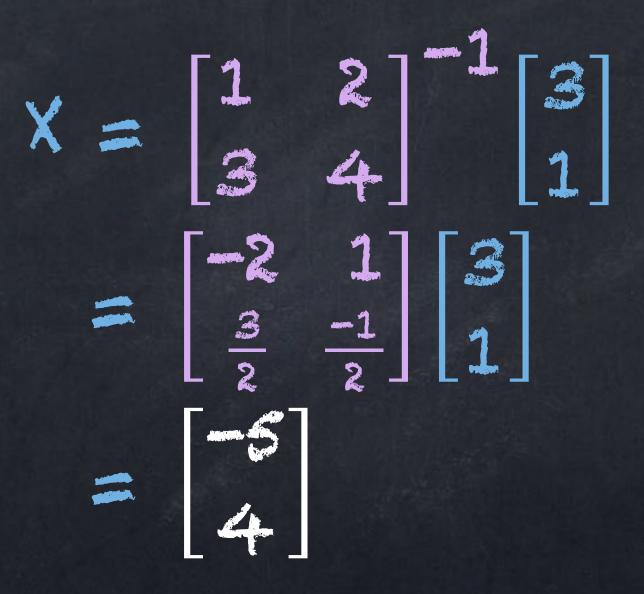
This has the format AX = B, so we can solve it by $A^{-1}AX = A^{-1}B$ $IX = A^{-1}B$ $X = A^{-1}B$

Note: we cannot say $X = BA^{-1}$ because $A^{-1}B$ is not the same as $A^{-1}B$.

matrix You need to be comfortable with calculations before solving equations.

Algebra

Task: Solve $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} X = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.





For 2×2 matrices, calculating the determinant is easy:

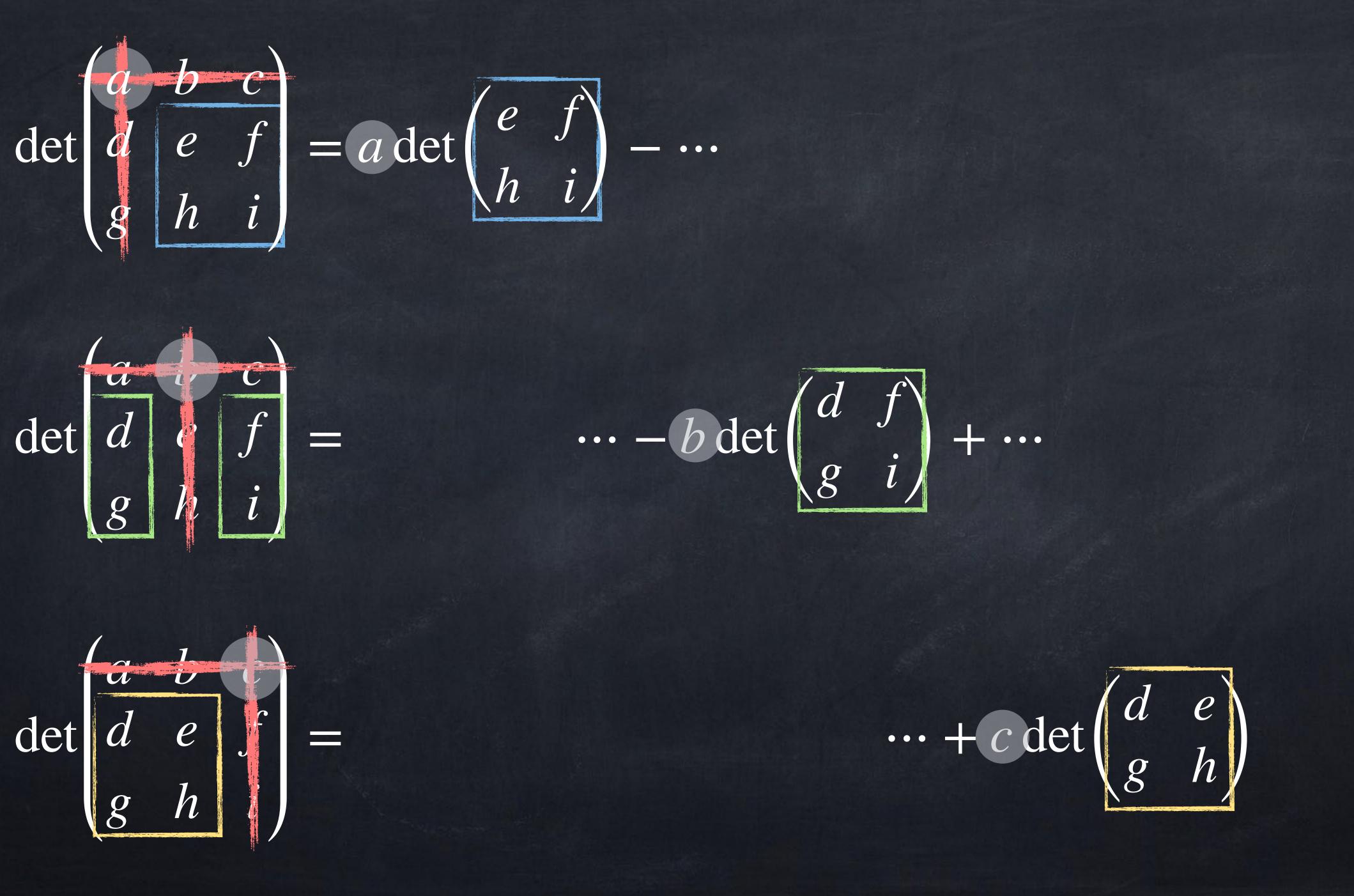
For 3×3 matrices, calculating the determinant is more work: $det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = a det \begin{pmatrix} e & f \\ h & i \end{pmatrix}$

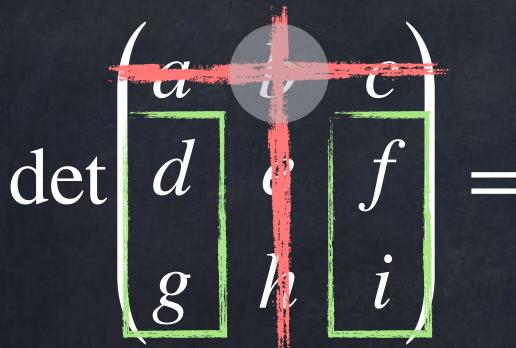
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$$det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc.$$

$$-b \det \begin{pmatrix} d & f \\ g & i \end{pmatrix} + c \det \begin{pmatrix} d & e \\ g & h \end{pmatrix}$$

There is a nice pattern to help you remember/use this formula...







Calculate det $\begin{pmatrix} 1 & -2 \\ 0 & 4 \end{pmatrix}$.

Calculate det $\begin{pmatrix} 3 & -2 \\ 3 & 4 \end{pmatrix}$.

Calculate det $\begin{pmatrix} 5 & 3 \\ p & 4 \end{pmatrix}$. = 20 - 3p



Calculate det $\begin{pmatrix} 5 & -1 & 0 \\ 3 & 1 & -2 \\ 3 & 0 & 4 \end{pmatrix}$. easy to forget $= 5 \operatorname{det} \begin{pmatrix} 1 & -2 \\ 0 & 4 \end{pmatrix} - \begin{pmatrix} -1 \\ -1 \end{pmatrix} \operatorname{det} \begin{pmatrix} 3 & -2 \\ 3 & 4 \end{pmatrix} + 0 \operatorname{det} \begin{pmatrix} 3 & 1 \\ 3 & 0 \end{pmatrix}$ = 6(4) - (-1)(18) + 0(-3)= 20 + 18 + 0 - 38

Calculate det $\begin{pmatrix} i & j & k \\ 3 & 1 & -2 \\ 3 & 0 & 4 \end{pmatrix}$.

Magic: vector cross product is exactly this! $[3, 1, -2] \times [3, 0, 4] = 4\hat{i} - 18\hat{j} - 3\hat{k}.$

 $= i det \begin{pmatrix} 1 & -2 \\ 0 & 4 \end{pmatrix} - j det \begin{pmatrix} 3 & -2 \\ 3 & 4 \end{pmatrix} + k det \begin{pmatrix} 3 & 1 \\ 3 & 0 \end{pmatrix}$